A Parameterized Family of Equilibrium Profiles for Three-Player Kuhn Poker

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One Slide Summary

- Kuhn Poker is introduced and solved, leading to advances in:
  - Strategy representations [Koller and Pfeffer, 1997]
  - Opponent modelling [Hoehn et al., AAAI 2005]
  - Equilibrium algorithms [Ganzfried and Sandholm, AAMAS 2010]
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  - Equilibrium algorithms [Ganzfried and Sandholm, AAMAS 2010]
- 3-player Kuhn Poker introduced, but not solved
- Here, we present a family of solutions for 3-player Kuhn Poker
Outline of Talk

- Motivation and Rules of Kuhn Poker
- Nash equilibrium in Kuhn Poker
- Nash equilibrium in 3-player Kuhn Poker
- Conclusions and Future Work
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Why Poker Research?

• Classic games, such as chess and checkers, are:
  - Deterministic
  - Binary outcomes (+ draw)
  - Perfect information

• On the other hand, poker contains
  - Stochastic events
  - Varying outcomes
  - Imperfect information
Kuhn Poker

[Kuhn 1953]
Kuhn Poker

Kuhn 1953
Kuhn Poker

[Kuhn 1953]
Kuhn Poker

Check?
Bet?
Kuhn Poker

Bet!

Fold?
Call?

[Kuhn 1953]
Kuhn Poker

Call.

[Kuhn 1953]
Kuhn Poker

[Kuhn 1953]
Kuhn Poker

-2  Lose.

+2  Win!

[Kuhn 1953]
3-Player Kuhn Poker

[Abou Risk and Szafron, AAMAS 2010]
3-Player Kuhn Poker

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3-Player Kuhn Poker

Fold.

[Abou Risk and Szafron, AAMAS 2010]
3-Player Kuhn Poker

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3-Player Kuhn Poker

Lose.

Lose.

Lose.

Win!

[Abou Risk and Szafron, AAMAS 2010]
Kuhn Poker Games

- Toy poker games
- Players can bluff, slow play
  - Strategic elements of Texas Hold'em
- Small enough to analyze by hand
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- Motivation and Rules of Kuhn Poker
- Nash equilibrium in Kuhn Poker
- Nash equilibrium in 3-player Kuhn Poker  
  New!
- Conclusions and Future Work
Definition of Nash Equilibrium

- **Example: Rock-Paper-Scissors**

```
    R  P  S
R  0  -1  +1
P  +1  0  -1
S  -1  +1  0
```

- “No one can change their strategy and do better”
  - assuming all other players' strategies are fixed
- Every game has at least one equilibrium [Nash 1950]
Equilibrium in Kuhn Poker

Bet: $p \in [0, 1]$  
Bet: 0  
Bet: $\frac{p}{3}$  

Slowplay ($p < 1$)  
Bluff ($p > 0$)  

[Kuhn 1953]
Equilibrium in Kuhn Poker

Mixed Strategy

Bet!

Call: \(\frac{1}{3}\)

Call: 1

Call: 0

[Kuhn 1953]
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Equilibrium in 3-Player Kuhn Poker

Bet: 0

Call: 1

Call: \( \frac{1}{2} \) if P3 folds
Call: 0 otherwise

Passive strategy

New!
Equilibrium in 3-Player Kuhn Poker

Check.

Bet: \(b_1\)  
Bet: \(b_2\)  
Bet: 0  
Bet: \(2b_1 + 2b_2\)

\[
\beta = \max \{b_1, b_2\} \leq \frac{1}{4}
\]
Equilibrium in 3-Player Kuhn Poker

Check.

Check.

Bet: \( p \in \left[ 0, \min \left\{ \frac{1}{2'}, \frac{2 - b_1}{3 + 2b_1 + 2b_2} \right\} \right] \)

Complicated

New!
Equilibrium in 3-Player Kuhn Poker

\[-\kappa \left( \frac{1}{2} + \beta \right)\]

\[\frac{\kappa}{2}\]

\[+ \kappa(1 + \beta)\]

\[\kappa = \frac{1}{24}\]

New!
Equilibrium in 3-Player Kuhn Poker

\[-\kappa \left( \frac{1}{2} + \beta \right) \]

\[\kappa = \frac{1}{24}\]

\[\kappa(1 + \beta)\]

Controls \(\beta\)!

\[\beta = \max \{\text{Bet: } \clubsuit \spadesuit, \text{ Bet: } \text{K} \heartsuit\}\]

New!
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Conclusions

- First analytically-derived 3-player Kuhn Poker Nash equilibrium profiles
  - Largest game with more than 2 players to be solved analytically
- Profiles exhibit ability for second player to transfer utility between the two opponents
- Proved certain Nash equilibrium strategies are robust
- Future work:
  - Other 3-player Kuhn Poker equilibria?
  - Can we build insights into other multiagent domains?
Thanks for Listening!

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